Can You Feel the Beat? Measuring the Quantum Beat

(Determining the Distinguishability between Two Photons)

by

Shivam Suthendran (20499543)

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ABSTRACT

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# CHAPTER 1

INTRODUCTION

What defines the quantum nature of a system? Is it particle size? Is it the inclusion of the Planck constant? Is it that the system is probabilistic? While these may contribute to the quantum nature of a system, one of the hallmarks of a quantum system is in the indistinguishability of particles. Indistinguishability, also called indiscernible or identical particles, is the consideration that one particle can not be distinguished from another. While it is not the only hallmark of a quantum system, indistinguishability is a purely quantum effect (1) and can be used to determine the nature of a system in question.

While the concept seems simple, it has profound effects, particularly in quantum optics. But before we get there, let us further elaborate on the concept of indistinguishability. In classical mechanics, while two particles may be identical in their inherent qualities, we are still able to identify one from another using their relative positions and momenta. Following this, we are then able to follow the subsequent trajectories and interactions of these particles. However, suppose we considered two electrons instead? What would the evolution of their collision look like?

|  |  |
| --- | --- |
|  |  |
| Fig 1.1 – Which is the correct path? | |

For two particles that are inherently not different from one another, which path seems more correct? Especially when taking Heisenberg’s uncertainty principle into consideration, a particle’s location ends up having very little meaning outside of the time of measurement. We now have no real way to distinguish the particles, and as such, we cannot follow particle trajectory in the same way as in the classical case.

What we have above seems rather abstract, more like a thought experiment, how could this possibly have any real-life applications? However, as mentioned above, the lack of indistinguishability can lead to some very interesting physical results. This leads us to the work done by Legero, Wilk, Kuhn, and Rempe. In 2003 they produced a paper detailing the mathematical framework that could demonstrate the distinguishability in photons. In particular, they found that if there was a difference in frequency between the photons, this would lead to an effect they called the quantum beat.

|  |
| --- |
|  |
| Fig 1.2 – The Beat Effect |

What is the quantum beat? When two photons, that are distinguishable due to a difference in frequency, and have a temporal delay with respect to one another that is shorter than their wave packet length, interact; they can create an interference effect that looks like a beat effect. This can be demonstrated experimentally by graphing coincidence counts of the output photons.

It is my intention to convince the reader that the presence of a quantum beat can be used to determine the distinguishability of photon sources. First, I must calculate the second order correlation function, I then take this result and integrate it over all possible detection times (τ), to determine the probability distribution of all possible detection times. This will be contrasted with the probability distribution of the Hong-Ou-Mandel experiment, as well as with Legero, Wilk, Kuhn, and Rempe’s work. However, I’m getting a bit ahead of myself, before we get there, in Chapter 2, I will provide historical and mathematical context of the experiment and the analytic techniques used in this thesis. In Chapter 3, I will provide my own contributions to the demonstration of the quantum beat, with a detailed analysis of my findings. Finally, in Chapter 4, I will make my conclusions with future considerations and improvements. It is my genuine hope that you will have learned as much as I have through the reading of this thesis.

# CHAPTER 2

CONTEXT

Before I can delve into the work that I have done over this project, it is important that I provide context for the work, this includes historical and technical context along with an analysis of present work. This is done in three parts:

1. The Framework of Quantum Information Processing and Quantum Optics
2. The Hong Ou Mandel Experiment
3. Hanbury-Brown Twiss Experiment

## 2.1 Quantum Background

One of the first things that needs to be established is the description of the object being tested, in this case, the photon. How does one quantify the photon, and quantify it in such a way that experimentation can be performed to an end result? We address this question by first drawing parallels between the quantum harmonic oscillator and the electron magnetic field. If this parallel is indeed possible, we must then seek a form that allows information about frequency to be accessible to us, as that will be the main mode of distinguishability in this paper. With this, we will have then established the mathematical description of the object in question – the photon.

### 2.1.1 Quantum Information Processing

First proposed by Paul Benioff in 1980, and built upon by Feynman in 1981, quantum computation takes on the framework of quantum mechanics to perform simulations that are far beyond the ability of classical computation. In particular, it takes advantage of superposition and entanglement; both of which are fundamental to quantum mechanics. Let us consider the superposition principle, and its use in quantum computation. Put simply, prior to measurement, a given quantum state is said to have access to and exists in multiple states. Upon measurement, a state is picked with some probability, and no information of its previous state is accessible.

Before we continue in more depth, let us consider the basic premise behind classical computation. The basic unit in classical computation is the bit, which takes on either a 0/1 value. Given these bits, logic gates can be applied to them, which serve to compare and/or combine the input bit values and provide an output bit in accordance with these gates and inputs.

In quantum computation, the basic unit of computation is the qubit. A qubit’s information is encoded in the following forms:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.1)  (2.1.2) |

This notation is called Dirac notation and equations 2.1.1 and 2.1.2 denote the ground and excited states, respectively. The qubit can exist as these states individually, or as a superposition of the two states. As stated previously, this means this exists as and has access to both the 0 and 1 states. This is represented in Dirac notation as:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.3) |

This is a normalised superposition of the ground and excited states. Each state has a complex coefficient (in this case ), the modulus square of which would return the probabilities of measuring each state (which in this case would be ). Thus, upon repeated measurement of the system we would be able to attain measurements for both the ground and excited states with a probability of . This elucidates the power of quantum computation over classical computation in its ability to produce all possible results over repeated iterations.

Another important reason that quantum computation supersedes classical computation is due to entanglement. We have thus far only considered single qubits, but suppose we consider multiple qubits instead. Like the single qubits, they exist as a superposition of basis states; however there exist states that can not be written as separable products of the individual states.

Consider a second qubit of the following form:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.4) |

We can write binary qubit state, the tensor product of the two states, is given as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.5) |

This is a separable state, as it can be written as a product state of the two qubits. The state is spanned by four basis states, of which each qubit brings two basis states.

A non-separable state is one that can not be decomposed into a product of the two qubits. There exists some correlation term that can not be cleanly divided between the two qubits. When the state has this form, it is said to be an entangled state. The Bell states are examples of non-separable states, and they represent all combinations of maximally entangled basis vectors. They are:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.6) |
|  |  | (2.1.7) |
|  |  | (2.1.8) |
|  |  | (2.1.9) |

### 2.1.2 States and Measurement

#### 2.1.2.1 Object Property Representation

I have loosely used the terms measurement and state but have yet to provide context for either. Suppose we were to consider an electron. What makes an electron an electron? What do we seek to know from the electron? Well, we can consider the properties of an electron, such as mass, angular momentum, spin etc. These properties give us information about the ‘state’ of an electron.

Mathematically, the set of all possible states within a property, spans a space called the Hilbert space. As an example, the set of all spin states of an electron spans a Hilbert space that is two-dimensional. This space has a set of vectors, called basis vectors, that span the entirety of the Hilbert space. As per the spin example, these basis vectors are the up and down spin states. One can generate any state in this Hilbert space as a linear combination of these basis states. Thus, a specific electron spin state is a vector within this Hilbert space, and as such, can be written as a linear combination of the up and down spin states.

In Dirac notation, these vector states are represented as ‘kets.’ Using our spin example again, the spin down state looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.10) |

We can see that this also has a column vector form. These column vectors can be transposed into row vectors, which belong to a corresponding space that is dual to the Hilbert space. These dual vectors are a set of functions that act upon our vectors and produce scalars. This is represented in Dirac notation via ‘bras,’ which look as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.11) |

This the bra corresponding to equation 2.1.10. A small, but important, note is to remember that the Hilbert space is a complex vector space, as such dual vectors are conjugate transposes of the vectors, rather than just the transposes.

#### 2.1.2.2 Accessing these Properties

Now that we know what states are how the represent properties of an object in question, how are we supposed extract this information for any use? Much like in the classical world, information is gained via measurement. The act of measurement is represented mathematically via an operator. Each basis vector in the Hilbert space is an eigenvector that has an associated eigenvalue. When an operator acts upon these vectors, it pulls out the associated eigenvalue. Again, using the spin analogy, suppose I wanted to measure the spin in the z direction:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.12) |

The operator acts on the ket to pull out the appropriate eigenvalue, . The set of all basis vectors form an orthonormal set, and when taking their inner product, we end up with a 0 if they are different, and a 1 if they are the same. In this case they are and thus we are left with our measurement:

In a superposition this will pull out a weighted measurement of the mean, suppose we have some state , which looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.13) |

Then taking the same measurement of the spin as in equation 2.1.12, gives us:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.14) |

This is also called the expectation value of the system and gives the weighted average measurement of an operator. Often the averages of a given operator are denoted as , wherein is the operator in question.

### 2.1.3 Quantum Harmonic Oscillator

I have used the ground and excited states quite liberally in the section 2.1.1; however, I have not defined the framework from which they come from. Much like in classical computation, quantum computation also utilises a two-level system. One way this occurs is by piggybacking off of the solutions for the quantum harmonic oscillator. We define this problem as follows:

|  |
| --- |
|  |
| Fig 2.1.1: Particle on a Spring |

Consider a typical problem in classical mechanics: a particle oscillating on a spring. We know that this has the following Hamiltonian:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.15) |

Wherein:

* is the particle momentum
* is the particle mass
* is the particle position
* And is the particle oscillation frequency

In order to make this a ‘quantum’ harmonic oscillator, I will quantize the variables. For the purposes of this paper, this will seem as though I’m simply putting hats on the variables while holding the canonical commutation relationship []= in mind. However, the mathematics is far more in depth and beyond the scope of this paper.

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.6) |

The typical approach is to solve this as a second order ordinary differential equation which will yield the following wavefunction:

|  |  |  |
| --- | --- | --- |
|  | ) | (2.1.17) |

Wherein:

* are the Hermite Polynomials and are defined as follows:

|  |  |  |
| --- | --- | --- |
|  | ) | (2.1.18) |

* defines the energy levels of the system

However, this is not quite what we’re looking for. In order to get a more suggestive form, we can consider a small redefinition of the variables, which was first conceived by Paul Dirac:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.19) |
|  |  | (2.1.20) |

These are called ladder operators and represent the action of lower or raising the energy level or the state. As an example, the raising operator can act on the ground state to given us the first excited state:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.21) |

Similarly, the lowering operator can take this excited state and lower it as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.22) |

One can take these operators and rewrite the Hamiltonian as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.23) |

Wherein the product of the ladder operators can be recast as , the number operator. This counts the number of excitations. For example, if the number operator were to act on the first excited state, it would count a single excitation:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.24) |

We have now established the general idea behind the quantum harmonic oscillator; but how does this relate to quantum optics? I seek to address this in the next section.

### 2.1.4 Quantum Optics

Of the multiple proposed modes for computation, one of the first and most popular is the photon. Information is encoded into the photon in multiple possible modes: frequency, spatial mode, polarisation, and time of arrival. This is a natural choice for candidacy due to the photon’s high durability (weak interactions with the environment) and high mobility. In this thesis, we will pay particular attention to the frequency mode of encoding. The harmonic oscillator framework is a natural fit for describing photon interactions such as excitation, absorption, and photon counting.

#### 2.1.4.1 Quantization of an Electromagnetic Field

However, we can not employ the harmonic oscillator convention as is. First, I must demonstrate that this framework is indeed a natural fit for the photon. To do this first consider the classical electromagnetic fields governed by Maxwell equations (in a vacuum):

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.25) |
|  |  | (2.1.26) |
|  |  | (2.1.27) |
|  |  | (2.1.28) |

Wherein:

* is the nabla/del operator
* is the electric field
* is the magnetic field
* is the speed of light within a vacuum

We can take the curl of the last two equations (equations 2.1.27 and 2.1.28), and when we do we find the following second order differential equations:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.29) |
|  |  | (2.1.30) |

The solutions for the electric field have the following form:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.31) |

Wherein:

* is a vector that denotes an arbitrary direction
* is the wave vector in the direction of wave propagation
* denotes the polarization of the vector, perpendicular the direction of propagation
* is the frequency of the wave, such that

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.32) |

* are spatial plane wave solutions of the form (also called mode functions):

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.33) |

* + With representing the electric field component with respect to polarization
  + representing the unit volume within which the field is confined
* and representing the complex amplitudes associated to each temporal mode

As these solutions are found with the assumption that the wave is propagating in a vacuum, there are no free charges. As such this implies that the polarization is orthogonal to the direction propagation.

Now to consider the magnetic field, which can be found using equation (2.1.21)

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.34) |

Wherein:

* are spatial plane wave solutions of the form (also called mode functions):

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.35) |

* + With representing the magnetic field component with respect to polarization and given by the following relationship:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.36) |

Given the electric and magnetic field equations define above, we can now determine the energy of a classical electromagnetic field as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.37) |
|  |  | (2.1.38) |

These look very close to quantum harmonic oscillator solutions, however there are some considerations before we can call them similar. As with the quantum harmonic oscillator, we must quantize the complex amplitudes. Again, this is more complex than what it seems (simply putting hats atop the operators and changing the star to a dagger); however, suffice it to understand that they are more complex than what is given. We must also consider that these operators obey bosonic commutation relations, ie:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.39) |

Our Hamiltonian then becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.40) |
|  |  | (2.1.41) |

Which takes advantage of the bosonic commutation relationships to get the final equality. We can safely conclude that the photon indeed follows a harmonic oscillator framework, with the careful consideration that ladder operators follow a bosonic commutation relationship. But how can we write it in such a way that takes frequency into consideration, rather than polarization.

#### 2.1.4.2 Temporal Mode Functions

The electromagnetic wave solutions are found with consideration to polarization, but as I mentioned before, the photon can encode in multiple modes. In this thesis we pay particular consideration to the frequency mode of encoding. As such, I can not use the polarization mode functions, and need to seek solutions associated to frequency instead. How do we go about this?

Mode functions were first characterized by Glauber and Titulær in 1966 and were proposed as a solution to the real-world problem that photon emissions are never exactly monochromatic, but instead a wave packets that hold multiple frequencies. They considered that the spectral width, the spectrum of frequencies contained by a photon, is roughly the inverse of the duration of the wave packet. This gave rise to the idea of a spatio-temporal wave packet, that was not characterized by polarization. As time and frequency are Fourier transforms of one another, they were then able to characterize the wave packet with respect to its frequency as well. This derivation will be done entirely without Fourier transforms, and as such will not be defined further in this thesis. Suffice it to say that there is an intrinsic relationship between time and frequency that allows for this transformation to occur.

We shall pay particular attention to defining temporal mode functions. Much like equation 2.1.31, we will assume a similar general form for the equation of an electric field (in the frequency domain) as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.42) |

We seek to create new creation and annihilation operators that will generate discrete photons which contain a collection of frequencies. Thus, we define a linear superposition of these and which are modified by a set of complete and orthogonal weight functions . This looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.43) |
|  |  | (2.1.44) |

These operators obey the bosonic commutation relationships:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.45) |

As mentioned above, these orthogonal weight functions, , are also complete. This means that they obey the following relationship:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.46) |

Using this, we can now define inverse relationships of the following form:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.47) |
|  |  | (2.1.48) |

We now have all the ingredients to create our temporal mode functions. These are defined using equations (2.1.34, 2.1.39, and 2.1.40) as:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.49) |
|  |  | (2.1.50) |

These temporal mode functions are key to the set up of the quantum beat problem, as they play the role of defining the ‘shape’ of the photon. All future calculations will be done in units of , or natural units for simplicity.

This thesis will use two types of temporal mode functions for photons as given by Wooley, Lang, Eichler, Wallraff, and Blais:

The Gaussian Photon:

Which has the following mode function:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.51) |

Wherein:

* denotes the pulse width
* represents the temporal offset between the pulses

representing photon a

* + representing photon b
* representing the difference between the frequencies
* Frequencies that are centered about
  + Thus representing photon a
  + And representing photon

The Gaussian mode functions defines an idealized wave packet. It is called a ‘packet’ as it consists of multiple wavenumbers clustered about a single value. This mathematically exemplifies ideal monochromatic emission.

The Lorentzian Photon:

Which has the following mode function:

|  |  |  |
| --- | --- | --- |
|  |  | (2.1.52) |

Wherein:

* denotes the decay rate of the pulse
* represents the temporal offset between the pulses
  + representing photon a
  + representing photon b
* representing the difference between the frequencies
* Frequencies that are centered about
  + Thus representing photon a
  + And representing photon
* denotes the unit step function

This shape arises in the case of homogenous broadening. In general, photon emissions are due to excitation and subsequent de-excitation of a quantum system. Each energy level of this quantum system has its own energy profile that is different from one energy level to the next. This profile is proportionate to the emission spectrum of the emitted photon upon de-excitation. The energy level is also proportionate to the natural lifetime of the excited state in question. The lifetime of the excited quantum system is not an exact model but exists upon a distribution about an average lifetime. We can also then infer that the energies, which are proportionate to lifetime also exist on a distribution and as such are broadened. The contribution of these effects on the photon are called homogenous broadening, and are better demonstrated via a Lorentzian curve, rather than a Gaussian.

~

In this section we have successfully defined the photon with respect to frequency under a quantum framework. We are now able to employ them for experimentation which will be further elaborated upon in the next section – The Hong, Ou, Mandel Experiment.

## 2.2 Hong, Ou, and Mandel Experiment

A landmark experiment in the field of quantum optics, the Hung-Ou-Mandel (HOM) effect was first demonstrated by Chung Ki Hong (홍정기), Zheyu Ou (区泽宇), and Leonard Mandel in 1987 at the University of Rochester. In this section we will dissect this experiment and put it under a mathematical framework that will make it accessible for our experimentation. We will begin with the experimental set up with a follow up discussion on the results and following analysis.

### 2.2.1 Experimental Set Up

Though this experiment is a hallmark in quantum optics, its execution is deceptively simple. It employs the use of a beam splitter upon which two photons are incident, the resultant photons and their detections are then analyzed. But I get ahead of myself, let me break this down a little further, starting with the beam splitter.

The beam splitter is an optical device that takes any input beam and divides it into two ‘arms.’ In most experiments, a beam splitter splits the beam into two constituent halves of the input beam, and as such is called a beam splitter. However, there are other beam splitters, such as the beam splitter, which splits the beam into and output arms. The beam splitter is generally a cube of glass which is made up of two glass prisms.

As mentioned above, there are two indistinguishable photons that are incident upon a beam splitter. Theoretically there are four possible outcomes for the output photons:

1. And photon a/b is reflected while photon b/a is transmitted (this has a multiplicity of 2) – as seen in figure 2.2.1.1, in the image labelled 1 and 4 respectively
2. Both input photons are transmitted – as seen in figure 2.2.1.1, in the image labelled 2
3. Both input photons are reflected – as seen in figure 2.2.1.1, in the image labelled 3

|  |
| --- |
|  |
| Fig. 2.2.1.1 – The Possible Beam Splitter Outputs |

This can be represented mathematically by first considering the photon generation. Suppose we call the input photons a and b respectively. Then their generation can be demonstrated via the application of photons’ creation operators. This looks as follows:

|  |  |  |
| --- | --- | --- |
|  | = | (2.2.1) |

These input photons will go through a beam splitter. This can be modelled as a unitary operator and looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.2.2) |

Wherein:

* and are the creation operators of the photons incident upon the beam splitter
* and are the creation operators of the photons output from the beam splitter

|  |
| --- |
| Sample Experiments – 2-Photon-Interference by Hong, Ou & Mandel – qutools |
| Figure 2.2.1.2 – The Experimental Set-Up |

The output photons were measured via two photodetectors on the other side of the beam splitter, which can be seen in figure 2.2.1.2. The temporal separation between the photons were varied as well. As the output photons were incident upon the beam splitter, the researchers measured the coincidence counts, that is to say that they measured a count when one photon was detected in each beam splitter at a given time. In the next section I will detail what they had found.

### 2.2.2 Results

|  |
| --- |
| Hong–Ou–Mandel effect - Wikipedia |
| Figure 2.2.1.2 – The HOM Dip |

What they found went against their expectations in that when the photons had no temporal overlap and were completely indistinguishable, no coincidence was measurable. That is to say that both photons were incident upon one photodetector or the other. This HOM dip as demonstrated, in figure 2.2.2.1, was deemed the experimental signature.

This result can be demonstrated mathematically by considering the transformation given in equation 2.2.2. We can rewrite the creation operators of photon a and b as a linear combination of the output creation operators of photons c and d. This looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2.2.3) |
|  |  | (2.2.3) |

We can then rewrite equation 2.2.1 as follows:

|  |  |  |
| --- | --- | --- |
|  | = | (2.2.4) |

Operators from two different Hilbert spaces commute, thus . As such we are left with:

|  |  |  |
| --- | --- | --- |
|  | = | (2.2.4) |

This gives us the exact result found in experiment, with no term to suggest a coincidence count.

~

We have now established the experimental framework for the experiment that will be the basis for my thesis. In the final section, we will be discussing the technique used to measure our system for usable results.

## 2.3 Hanbury-Brown Twiss Experiment

Before we continue with quantum optics, we will take a brief foray into astronomy to the work of Robert Hanbury-Brown and Richard Q. Twiss. This will provide us with the last piece of the puzzle – the analytic method – which will help us understand the results that we generate.

### 2.3.1 Historical Context

The primary mode of determining the diameter of stars was the Michelson interferometer. The general idea behind the Michelson interferometer was the collection of light from a single source upon two mirrors, which were separated by some distance ‘*d.*’ This light was then focussed upon the telescope and onto a focal plane. If the light was coherent, then an interference pattern would be formed. As the distance between the mirrors was changed, so too would the interference pattern. One could determine the angular size of the star in question, by studying the change in the fringe pattern relative to the distance between the mirrors.

This idea falls apart when the distance between the mirrors is too large and any small disturbance to the collection mirrors affects the interference fringes and prevents any viable measurements from being made. Cue Hanbury-Brown and Twiss, who proposed a simpler arrangement. Let’s cut the mirrors out of equation all together and simply focus the light directly onto two photomultipliers, without needing to generate an interference pattern at all. Instead the correlations between the two generated photocurrents were used to determine star size instead.

Hanbury-Brown and Twiss were met with vehement skepticism and criticism. As such they demonstrated the validity of their idea with what is now known as the Hanbury-Brown-Twiss (HBT) experiment, which we will discuss further in this next section.

### 2.3.2 The Experiment and the Correlation Function

Hanbury-Brown and Twiss decided to develop a proof of concept to demonstrate the validity of their interferometer on a smaller scale, with the use of a mercury lamp. The light from this incident upon a half-silvered mirror, which acts like a beam splitter, and splits the incident beam into two constituent parts. Each output beam is then amplified by a photomultiplier and an AC-coupled amplifier, which gave outputs proportional the fluctuations in current. One photocurrent was delayed by a time by passing it through a time delay generator. The two signals are then integrated and averaged over a given time period. As the resultant current is proportional to the input beam intensity, we can assume that current fluctuations will also be proportional to intensity fluctuations. Thus, we have that:

|  |  |  |
| --- | --- | --- |
|  |  | (2.3.1) |

We have the general idea of how to measure this? But how does one mathematically demonstrate a fluctuation? How can we demonstrate the output of the two signals in a quantifiable way? This question brings us to the introduction second-order correlation function, which allows us to glean information about the outputs of the experiment. This is given by the following equation:

|  |  |  |
| --- | --- | --- |
|  |  | (2.3.2) |

Which is indeed similar to the equation 2.3.1.

How does this relate to quantum optics? We don’t have photocurrents, or intensities, but rather individual photons instead. As the number of photons increase, the more intense the light source becomes, thus we can draw a proportionality between he number of photons and intensity. This relationship lends itself to the following redefinition of :

|  |  |  |
| --- | --- | --- |
|  |  | (2.3.3) |

The results from this equation can be classified neatly in figure 2.3.2.1 as follows:

|  |
| --- |
|  |
| Figure 2.3.2.1 – Results of the Second Order Correlation Function |

With this redefinition, we now have our final piece to the experimental puzzle, a way to understand our results in a quantifiable way.

~

With this final section, I have set up the experimental building blocks for my thesis. I have established a mathematical framework for the object being measured, the photon, in section 2.1. In section 2.2, I elaborated upon the experimental technique that the photons will be subject to. And finally, in section 2.3, I outlined the technique used to quantify the results of the experiment. In the next chapter I seek to employ these blocks to build an experiment that will help me determine the distinguishability of the photons emitted from a given source.

# CHAPTER 3

METHODOLOGY AND RESULTS

In chapter two we established the historical context and mathematical framework behind the experiment and analysis for the question we seek to posit: Can we determine if two photons are distinguishable from the quantum beat? In this chapter I seek to employ what we have learned to determine if that is the case. Legero, Wilk, Kuhn, and Rempe demonstrated the quantum beat by graphing the two-photon coincidence probability against the detection time. In this thesis I seek to demonstrate the same effect after integrating the second order correlation function over all possible initial detection times and secondary detection times. This will give me a probability distribution that I will use to confirm the presence of the Hung-Ou-Mandel dip, and also serve as a visual check for the quantum beat. Our first step is to construct the second order correlation function for the experiment in question.

## 3.1 The Second-Order Correlation Function – HOM Edition

We have from equation 2.3.3 the second order correlation function as follows, however in this case we are particularly interested in the correlations between the output photons. Thus, we will amend accordingly:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1.1) |

We can recast this as the product of their respective creation and annihilation operators. Before doing so, we can consider the subtlety that operators belonging to two different Hilbert spaces will commute. As such, we have that []= []=0. This allows us to rewrite what we have in the normal ordering convention, such that the creation operators are written to the left and the annihilation operators are written to the right. This looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1.2) |

However, we do not know how/what these output photons look like. So how do we progress? Recall from chapter 2, equation 2.2.2, this outlines the transformation of the incident photons into the emitted photons. While we may not know what photons c and d look like, we do know what incident photons a and b look like. Using equation 2.2.2, we can rewrite 3.2 with respect to these input photons a and b. The resulting equation has a total of 16 terms; however, we can simplify this by making the following considerations. The first is to recognize that we can ignore any terms that contain the square of the creation or annihilation operator, this is due to the consideration that we are only generating single photons. This eliminates 12 terms, leaving us with 4. The resultant function looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1.3) |

This still looks rather intimidating; thus we make our second consideration: The actual form of these time dependent creation and annihilation operators. As we are dealing with photons, they are not simply the creation and annihilation operators, but are instead modified by scalar mode functions. In particular, we are dealing with temporal mode functions, which for now we will simply denote as We will specify the exact functions in the next section. Thus we are able to separate the time dependence from the operators to look as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1.4) |

Our third consideration: As we are calculating expectation values, we must outline the state that we are using to calculate them. As mentioned in chapter two, the HOM experiment begins with two photons, a and b, being produced and incident upon the beam splitter, as such this will be the state upon which our calculations are performed:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1.5) |

Our final consideration is for the denominator, as there will always be a photon output from the beam splitter, the number operator of the outputs will always be 1, for the excitation in each output mode. Thus the denominator will simplify to 1.

When we combine the final three considerations, we end up with the following simplified expression, that will be used for the remainder of this thesis:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1.6) |

We now have the final form of the that will be used in the next section to calculate probability distributions for two different photons – the Gaussian and the Lorentzian photons.

## 3.2 Probability Distribution and Detection of the Quantum Beat

Now that we have established the correlation function associated to the HOM experiment. In this section, I seek to use it for two different photon distributions - The Gaussian and the Lorentzian. First, I will determine if oscillates with respect to the detection time *;* this will give me confirmation of the presence of the quantum beat. However, this is not sufficient to determine distinguishability. The , much like an expectation value, only gives me the average detection time. If I want to determine if a given detection time has a certain probability of occurrence, I must consider the entire distribution, not just the mean. In order to do this, I will integrate the second-order correlation function twice, once over all first photon detection times () and then over all possible detection time delays (). This gives me a distribution, of all possible photon measurements, and thus the likelihood of simultaneously detecting photons in both output photodetectors. I seek a change in the HOM dip in such a way that a photon is detectable in both photodetectors. If this occurs then I can safely confirm that this experiment can be used as a test to determine the distinguishability of photons.

### 3.2.1 The Gaussian Photon

Consider a Gaussian photon, previously described in equation 2.1.51, it has the following temporal mode function:

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.1) |

The associated , is then:

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.2) |

I can graph this to see how the photons will interfere with one another. However, just looking at this function I can see that the function oscillates with respect to detection delay times , suggesting that we are able to detect the quantum beat. In figure 3.2.1, I demonstrate that over multiple frequency differences,= 0, 5 and 10, that I’m able to detect this beating effect

|  |  |  |
| --- | --- | --- |
| a | b | c |
| Fig 3.2.1 – Second Order Correlation Function for Two Gaussian Photons – The blue axis representing the correlation between the two photons the red axis representing the detection time difference and the green axis representing the temporal separation between photons timefor frequency difference of a) 0 b) 5 and c) 10. | | |

We can find the associated probability distribution by first integrating this over all first photon detection times ().

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.2) |

Notice that for non-zero detection time delay (i.e. With the final probability distribution found when we integrate what we have above with the detection time delays (). This looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.3) |

Which corresponds to the results found by Woolley, Lang, Eichler, Wallraff, and Blaise, with mathematics computed using wolfram alpha.

Suppose we consider the simple case where there is no difference in frequency, ie =0 and. We then find the experimental signature of the HOM experiment. That is to say that if there is no delay between the photons (i.e. =0), then the probability of finding a photon in both photodetectors is 0.

We can graph this probability distribution with respect to the temporal separation between photons as follows:

|  |  |  |
| --- | --- | --- |
| a | b | c |
| Fig 3.2.2 – Probability Distribution for Two Gaussian Photons with a Pulse Width of 2 – Blue axis representing the probability P, the green axis representing frequency difference and the red axis representing the detection time separation a) Demonstrating the probability distribution with respect to and b) The level curves of probability with respect to frequency difference. c) The level curves of probability with respect to temporal separation. | | |

We can see the classic HOM dip in this simulation, but we notice that as the frequency difference between the two photons increases, the dip slowly rises, and as such there is an increase in the probability of attaining a coincidence count. This rising of the HOM dip would then serve to determine if the photons are distinguishable in the frequency domain.

Thus we can confirm that the photons are indeed distinguishable on two fronts, both in the presence of the quantum beat effect in the second-order correlation functions. We also determined that there is a in the rising of the HOM dip as the frequency increases, suggesting detection in both photodetectors. Let us consider the more realistic Lorentzian photon to see if I can see similar results.

### 3.2.2 The Lorentzian Photon

Now consider the Lorentzian photon, previously described in equation 2.1.51, it has the following temporal mode function:

|  |  |
| --- | --- |
|  | (3.2.) |

The associated , is then:

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.2) |

We see that the second order correlation function is indeed oscillatory, thus for non-zero detection time delay (i.e. we would expect interference between the two photons giving rise to a beat effect. This can be seen in figure 3.2.2 for a frequency difference of 2.

|  |  |  |
| --- | --- | --- |
| aChart  Description automatically generated with medium confidence | Chart  Description automatically generatedb | Chart, surface chart  Description automatically generatedc |
| Fig 3.2.1 – Second Order Correlation Function for Two Gaussian Photons – The blue axis representing the correlation between the two photons the red axis representing the detection time difference and the green axis representing the temporal separation between photons timefor frequency difference of a) 0 b) 5 and c) 10. | | |

We can find the associated probability distribution by first integrating this over all first photon detection times ().

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.2) |

Notice that for non-zero detection time delay (i.e. With the final probability distribution found when we integrate what we have above with the detection time delays (). This looks as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (3.2.3) |

Which corresponds to the results found by Woolley, Lang, Eichler, Wallraff, and Blaise, with mathematics computed using wolfram alpha.

Suppose we consider the simple case where there is no difference in frequency, ie =0 and. We then find the experimental signature of the HOM experiment. That is to say that if there is no delay between the photons (i.e. =0), then the probability of finding a photon in both photodetectors is 0.

We can graph this probability distribution with respect to the temporal separation between photons as follows:

|  |  |  |
| --- | --- | --- |
| a | Chart  Description automatically generatedb | Diagram  Description automatically generatedc |
| Fig 3.2.2 – Probability Distribution for Two Gaussian Photons with a Pulse Width of 2 – Blue axis representing the probability P, the green axis representing frequency difference and the red axis representing the detection time separation a) Demonstrating the probability distribution with respect to and b) The level curves of probability with respect to frequency difference. c) The level curves of probability with respect to temporal separation. | | |

We can see the classic HOM dip in this simulation, but we notice that as the frequency difference between the two photons increases, the dip slowly rises, and as such there is an increase in the probability of attaining a coincidence count. This rising of the HOM dip would then serve to determine if the photons are distinguishable in the frequency domain.

Thus we can confirm that the photons are indeed distinguishable on two fronts, the first is the presence of the beat pattern when the second order correlation functions, and the second is in the rising of the HOM dip as the frequency increases, making the photons more distinguishable. I can safely assume that I am able to determine the distinguishability of photons on the frequency basis, through the second order correlation function and the resultant probability distribution.

~

# CHAPTER 4

CONCLUSIONS AND FUTURE CONSIDERATIONRS

It’s the final brain cell